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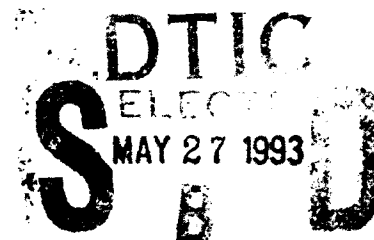
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Monterey, California



THESIS

REALIGNING THE
U.S. NAVY RECRUITING COMMAND

by

Garry S. Schwartz

March 1993

Thesis Advisor:
Second Reader:

Siriphong Lawphongpanich
Richard E. Rosenthal

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93-11904



93 5 20 09 9

REPORT DOCUMENTATION PAGE			Form Approved OMB No. 0704-0188	
<small>Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.</small>				
1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE March 1993	3. REPORT TYPE AND DATES COVERED Master's Thesis		
4. TITLE AND SUBTITLE REALIGNING THE U.S. NAVY RECRUITING COMMAND		5. FUNDING NUMBERS		
6. AUTHOR(S) SCHWARTZ, Garry S.				
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Naval Postgraduate School Monterey, CA 93943-5000		8. PERFORMING ORGANIZATION REPORT NUMBER		
9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)		10. SPONSORING / MONITORING AGENCY REPORT NUMBER		
11. SUPPLEMENTARY NOTES The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the U.S. Government.				
12a. DISTRIBUTION / AVAILABILITY STATEMENT Approved for public release, distribution is unlimited.		12b. DISTRIBUTION CODE		
13. ABSTRACT (Maximum 200 words) <p>This thesis addresses two problems in aligning the recruiting structure for Navy Recruiting Command. The first problem involves two decisions affecting recruiting stations within a single recruiting district: which stations should remain open and how many recruiters should be assigned to each open station? The second problem is to decide how many recruiters and stations each district should have.</p> <p>The first problem is formulated as a nonlinear mixed integer programming problem. To obtain a solution with readily available software, the problem is decomposed into four subproblems that are solved sequentially. This decomposition approach is empirically shown to yield near optimal solutions for problems of varied sizes. The second problem is formulated as a nonlinear resource allocation problem in which the objective function is not expressible in closed form. To efficiently solve this problem, the function is approximated in a piecewise linear fashion using the results from the first problem. To illustrate the applications of these optimization models, solutions were obtained for Navy Recruiting District Boston and Navy Recruiting Area 1, which consists of Albany, Boston, Buffalo, New York, Harrisburg, Philadelphia, Pittsburgh and New Jersey districts.</p>				
14. SUBJECT TERMS Mixed Integer Nonlinear Programming, Facility Location, Piecewise Approximating Large Mathematical Models, Decomposition of Mixed Integer Nonlinear Problems, Resource Allocation, GAMS.			15. NUMBER OF PAGES 57	
			16. PRICE CODE	
17. SECURITY CLASSIFICATION OF REPORT Unclassified	18. SECURITY CLASSIFICATION OF THIS PAGE Unclassified	19. SECURITY CLASSIFICATION OF ABSTRACT Unclassified	20. LIMITATION OF ABSTRACT UL	

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REALIGNING THE
U.S. NAVY RECRUITING COMMAND

by

Garry S. Schwartz
Captain, United States Marine Corps
B.S., Illinois Institute of Technology, 1985

Submitted in partial fulfillment
of the requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

NAVAL POSTGRADUATE SCHOOL

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ABSTRACT

This thesis addresses two problems in aligning the recruiting structure for the Navy Recruiting Command. The first problem involves two decisions affecting recruiting stations within a single recruiting district: which stations should remain open and how many recruiters should be assigned to each open station? The second problem is to decide how many recruiters and stations each district should have.

The first problem is formulated as a nonlinear mixed integer programming problem. To obtain a solution with readily available software, the problem is decomposed into four subproblems that are solved sequentially. This decomposition approach is empirically shown to yield near optimal solutions for problems of varied sizes. The second problem is formulated as a nonlinear resource allocation problem in which the objective function is not expressible in closed form. To efficiently solve this problem, the function is approximated in a piecewise linear fashion using the results from the first problem. To illustrate the applications of these optimization models, solutions were obtained for Navy Recruiting District Boston and Navy Recruiting Area 1, which consists of Albany, Boston, Buffalo, New York, Harrisburg, Philadelphia, Pittsburgh and New Jersey districts.

THESIS DISCLAIMER

The reader is cautioned that computer programs developed in this research may not have been exercised for all cases of interest. While effort has been made, within the time available, to ensure that the programs are free of computational and logic errors, they cannot be considered validated. Any application of these programs without additional verification is at the risk of the user.

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I. INTRODUCTION

A. BACKGROUND

Although the U.S. Navy is becoming smaller, the Navy Recruiting Command must continue to recruit in order to replace sailors at the lowest grade who are promoted or lost to attrition. Military downsizing means fewer recruits are needed, but these recruits are more difficult to obtain. This is due to several factors. First is the decrease in the recruiting budget and the rise in operating costs. Second is the emphasis on obtaining a greater proportion of quality recruits who are capable of manning ever more sophisticated weapons, sensory and communications systems. Recruiting these quality individuals is costly because they are in demand from other branches of the military, private industry and academia. Finally, the last factor is the expected competition from the National Service Corps that President Clinton promoted during his campaign. Many individuals in the prime target population for recruiting would likely find this program attractive, for it would be less restrictive and involve less risk than military service. In addition, it would keep people closer to families and friends, and would offer attractive educational benefits.

To assist Navy Recruiting Command in accomplishing this increasingly difficult task, this thesis addresses the problem of aligning recruiting stations and districts. Proper alignment of these organizational units would allow the Command to recruit more effectively.

B. CURRENT ORGANIZATION

The Navy Recruiting Command (NRC), located in Arlington, Virginia, is the headquarters for a nationwide network of recruiters. The command is organized into five recruiting areas with each area subdivided into districts. There is a total of 41 districts. Each of which is organized into zones which are composed of stations. The stations serve as the base from which the recruiters operate. In 1991, there were 1,283 Navy Recruiting Stations in the Continental United States. Recruiters actively pursue prospective recruits in the territories of their assigned station. In general, the territory of a recruiting station consists of a collection of adjacent zip code areas.

C. PROBLEM STATEMENTS

The problem currently faced by NRC is how to maintain an effective and efficient recruiting structure with a smaller budget. In this thesis, two problems are considered. One concerns the recruiting stations within the territory of a

single district and the other concerns the allocation of recruiting resources to several districts.

The first problem is to determine, for a given district, which stations from a list of candidates to open and the number of recruiters to assign to each open station. This problem is formulated as a nonlinear integer programming problem and it is called the *Location-Allocation Problem*. As is well known in operations research, these problems are difficult to solve. However, this thesis obtains near optimal solutions to the problem via a decomposition technique.

The second problem involves the allocation of recruiting resources to several districts. For example, if the current operating budget only supports 500 stations and 1000 recruiters, then how many stations and recruiters should be assigned to each district? This problem is formulated as a nonlinear resource allocation problem and it is called the *Multidistrict Allocation Problem*. The objective function of this problem is not expressible in closed form. To solve it, this thesis provides a methodology to obtain solutions through approximation using piecewise linear functions.

D. THESIS OUTLINE

The two optimization models mentioned above use results from forecasting models developed by analysts at NRC. For completeness, these models are described in Chapter II. Chapter III describes and formulates the location-allocation

problem. Chapter IV discusses the technique to decompose the problem in Chapter III. Chapter V provides a bounding technique which validates the claim that the decomposition technique provides near optimal solutions. Chapter VI describes the multidistrict allocation problem and presents a solution approach. Finally, Chapter VII summarizes the thesis and suggests areas for future research.

II. FORECASTING ENLISTMENT CONTRACTS

An extremely important input required by optimization models for locating recruiting stations and allocating recruiters is the ability to forecast the distribution of the target market across the continental United States (CONUS). Despite the increasing role of women in the military, the prime target market for military service continues to be males between 17 and 21 years of age with no prior military service. From this market, the Navy is primarily interested in enlisting *quality* recruits. The first section in this chapter presents NRC's definition of 'quality'. The next section describes four statistical models developed by NRC to forecast the distribution of quality contracts across CONUS. Finally, the last section discusses mathematical implications of these statistical models when embedded in the optimization models in the following chapter. These optimization models work equally well with different statistical models. The NRC models are selected mainly because of their accessibility and familiarity to NRC analysts.

A. QUALITY CONTRACTS

The Navy groups prospective recruits into major categories or cells based on two factors: educational attainment and mental aptitude. For the educational factor, there are only

two classifications. The first consists of prospective recruits who already possess a high school diploma or are currently high school seniors expecting to graduate. The second consists of those who are currently without a diploma and do not expect to obtain one in the future. The mental aptitude classification is determined by the score from the Armed Forces Vocational Aptitude Battery (ASVAB) or Armed Forces Qualification Test (AFQT). Each prospective recruit must take the ASVAB prior to induction into the Navy. The test measures arithmetic reasoning, word knowledge, general science, mechanical comprehension, and other skills. On the basis of the ASVAB scores, recruits are divided into eight mental groups, based on percentiles as indicated in Figure 1.

RECRUIT QUALITY MATRIX					
Armed Forces Qualification Test	Percentile	Mental Group	HIGH SCHOOL Diploma Graduate	NON HIGH SCHOOL Diploma Graduate	
	99	I	A	B	
	93	II			
	65	IIIU			
	50	IIIL	Cu	D	
	31	IVa			
	21	IVb	NOT BEST QUALIFIED		
	16	IVc			
	10	V	INELIGIBLE		
	0				

Figure 1: Recruit Quality Classification Matrix

Figure 1 displays the complete classification matrix of prospective recruits. The Navy regards a prospect as a quality recruit if the individual scores in the upper 50th percentile on the ASVAB test, categories I to IIIU, and has or is expected to have a high school diploma. Quality recruits are also referred to as A-Cell contracts.

B. FORECASTING A-CELL CONTRACTS

In a report by Bohn and Schmitz [Ref. 1], NRC developed four statistical models to forecast the number of A-Cell contracts in each zip code area of CONUS. These models include the following variables as predictors:

1. The **target population** of 17-21 year old males in the zip code: NRC estimated the population from the Navy's Standardized Territory Evaluation and Analysis for Management (STEAM) database for 1991.
2. The **recruiter share** assigned to each zip code: NRC assumes that a fractional number of recruiters or a *recruiter share* is assigned to each zip code. This fraction represents the amount of time recruiters devote to a particular zip code.
The optimization models in this thesis allocate recruiter shares to zip codes in order to maximize total production of A-Cells. In the statistical models below, NRC simply assumes that recruiter shares are proportional to the 1991 population of 17-21 year old males since no record of recruiter shares was maintained for each zip code.
3. The **distance** from the centroid of the zip code to its affiliated recruiting station: NRC used the longitude and latitude of the zip code centroids from commercially available software called MAPINFO [Ref. 2] to calculate the distances. In addition, NRC also assumes that the longitude and latitude of a recruiting station

are those of the centroid of the zip code in which the station is located.

4. The 1991 **population density** of the zip code: These densities are calculated from the physical zip code areas available from Litton Computer Services.

The following describes the four forecasting models developed by NRC. The first three are log-linear and the last is quadratic.

Log-Linear Regression Models

Model 1:

$$A_{cells_z} = a P_z^\alpha R_z^\beta T_z^\gamma D_z^\delta \quad \text{with } R^2 = 0.53033$$

where

P_z = the population of 17-21 year old males in zip code z

R_z = the recruiter share assigned to zip code z

T_z = the distance in miles from zip code z to its affiliated station

D_z = the population density in zip code z

$a = 0.08727, \alpha = 0.64179, \beta = 0.28405, \gamma = -0.32231,$

$\delta = -0.11923.$

Model 2:

$$A_{cells_z} = a P_z^\alpha R_z^\beta T_z^\gamma D_z^\delta \prod_{i=3,5,7,8} e^{k_i s_i} \quad \text{with } R^2 = 0.53229$$

where

$$s_i = \begin{cases} 1, & \text{if station is in Area } i \\ 0, & \text{otherwise} \end{cases}, \quad i = 3, 5, 7, 8$$

$a = 0.08401, \alpha = 0.64506, \beta = 0.27904, \gamma = -0.31380,$

$\delta = -0.11306,$

$$k_3 = 0.83109, k_5 = 1.03919, k_7 = 0.96295, k_8 = 1.15565.$$

Model 3:

$$A_{cells_z} = a P_z^\alpha R_z^\beta T_z^\gamma D_z^\delta \prod_{i=2}^6 e^{k_i s_i} \quad \text{with } R^2 = 0.53063$$

where

$$s_i = \begin{cases} 1, & \text{if \#recruiters} = i \\ 0, & \text{otherwise} \end{cases}, \quad i = 2, 3, 4, 5$$

$$s_6 = \begin{cases} 1, & \text{if \#recruiters} \geq 6 \\ 0, & \text{otherwise} \end{cases}$$

$$a = 0.05637, \alpha = 0.69148, \beta = 0.23419, \gamma = -0.33103,$$

$$\delta = -0.12732$$

$$k_2 = 1.05817, k_3 = 1.06999, k_4 = 1.15118, k_5 = 1.09317,$$

$$k_6 = 1.17982.$$

Quadratic Regression Model

Model 4:

$$A_{cells_z} = a + \beta R_z + \gamma T_z + \delta D_z + \epsilon T_z^2 + \zeta R_z T_z \quad \text{with } R^2 = 0.60598$$

where

$$a = 0.51845, \alpha = 0.00160, \beta = 3.51236, \gamma = -0.01095,$$

$$\delta = -0.00172, \epsilon = 4.66E-5, \zeta = -0.01798.$$

C. CONCAVITY ANALYSIS OF NRC PRODUCTION FUNCTIONS

In economic theory, mathematical functions that relate the amount of inputs into a process to a specific level of output are known as econometric production functions. Clearly the NRC models are such functions, since they relate the expected

number of A-Cell contracts produced to given amounts of inputs, such as recruiter share and target market population. An important class of production functions is the Cobb-Douglas function [Ref. 3] which has the form

$$f(v_1, \dots, v_n) = A v_1^{\alpha_1} v_2^{\alpha_2} \dots v_n^{\alpha_n} \quad (2.1)$$

where the function value $f(v_1, \dots, v_n)$ represents the output of the process due to the n inputs denoted by v_1, \dots, v_n . By definition, the v_i 's are generally assumed to be nonnegative. However, when α_i is negative, v_i must be positive. To ensure that nonnegative inputs yield nonnegative outputs, A is generally a positive constant.

The Cobb-Douglas function is homogeneous of degree $k = \alpha_1 + \alpha_2 + \dots + \alpha_n$ [Ref. 4] because

$$f(tv_1, tv_2, \dots, tv_n) = t^k f(v_1, v_2, \dots, v_n) \quad \forall t \geq 0. \quad (2.2)$$

In classical applications, the Cobb-Douglas functions also satisfies the conditions that (i) $k < 1$ and (ii) $\alpha_i \geq 0$ for all $i = 1, \dots, n$ [Ref. 5]. This implies that the function is decreasing return to scale, i.e., doubling all inputs results in less than doubled output. Furthermore, it can be shown that a Cobb-Douglas function satisfying the two conditions is strictly concave over the region where $v_i \geq 0$, $i = 1, \dots, n$ [Ref. 6]. The concavity plays an important role in optimization problems, for it guarantees global optimality.

The first regression model considered by NRC, Model 1, is similar in form to the Cobb-Douglas function. By redefining some variables, the function in Model 1 can be transformed into a Cobb-Douglas function in terms of the variables R (recruiter share) and T (distance from each zip code to its affiliated station). In particular, the function is rewritten as

$$f(R, T) = cR^{\beta}T^{\gamma} \quad (2.3)$$

where c is the positive constant $aP^{\alpha}D^{\nu}$. As stated, $f(R, T)$ is not concave in R and T because γ is negative. However, by defining ω as $1/T$, $f(R, T)$ can be written as

$$f(R, \omega) = cR^{\beta}\omega^{\lambda} \quad (2.4)$$

where $\lambda = -\gamma$. Since $\beta + \lambda < 1$ for Model 1, $f(R, \omega)$ is strictly concave. Similar analysis can be performed for Models 2 and 3 as well.

In the above transformation, $f(R, \omega)$ is undefined when $T = 0$. To avoid the problem of zero division, NRC assumes that the minimum distance between a zip code and its affiliated station is 0.5 miles, i.e., $T \geq 0.5$.

Model 4 is based on a quadratic function whose Hessian with respect to R and T is given by

$$H(x) = \begin{pmatrix} 0 & \xi \\ \xi & 2\epsilon \end{pmatrix}. \quad (2.5)$$

The eigenvalues of $\mathbf{H}(\mathbf{x})$ are

$$\theta_1 = \sqrt{\xi^2 + \epsilon^2} + \epsilon \text{ and } \theta_2 = -\sqrt{\xi^2 + \epsilon^2} + \epsilon. \quad (2.6)$$

Since $\epsilon = 4.66\text{E-}5$ and $\xi = -0.01798$, $\theta_1 > 0$ and $\theta_2 < 0$, i.e., $\mathbf{H}(\mathbf{x})$ is an indefinite matrix. Therefore, Model 4 does not yield a concave production function. [Ref. 7]

III. OPTIMAL STATION ALIGNMENT FOR ONE RECRUITING DISTRICT

The problem of aligning Navy recruiting stations within a single district involves selecting (or locating) recruiting stations to remain open and allocating recruiters to each of the open stations. The objective is to maximize the A-Cell contract production. A preliminary discussion of this problem was provided in an interim report to NRC by Lawphongpanich, Rosenthal and Schwartz [Ref. 8].

A. RELATED RESEARCH

Doll [Ref. 9] recently solved the problem of locating Marine Corps recruiting stations. His objective is to maximize potential Marine Corps accessions as measured by a factor called the Propensity Weighted Qualified Military Available. In addition, Doll addressed the problem at the county level; i.e., he assumed that CONUS is a collection of counties, and the decision is whether or not to locate a recruiting station in a given county. In a somewhat different problem, Celski [Ref. 10] used an uncapacitated facility location model to determine the optimal location of Army recruiting company headquarters. Celski's objective was to centrally locate a specified number of headquarters with respect to distance and population density of the target

market while simultaneously maximizing equity of recruiting responsibility.

B. LOCATION-ALLOCATION PROBLEM FORMULATION

To solve the station alignment problem in a given district, there are four sets of decisions to be made. The first set of decisions is to choose a subset of recruiting stations to open from a list of candidate stations. These candidates include existing and proposed stations. The second is to select the territory of each open station by assigning zip codes to stations. The third is to determine the number of recruiters to assign to each of the open stations. Finally, the fourth is to decide how to distribute the recruiter effort at a given station to zip codes in its territory. The main constraints are the number of stations to remain open and the number of available recruiters. The objective in aligning the stations is to maximize total A-Cell contract production in the district which can be predicted by the first log-linear regression model in Chapter II. (The formulations involving the second and third log-linear models are similar.) The mathematical formulation of the station alignment problem is called the location-allocation (LOCAL) problem and it is stated below.

C. MATHEMATICAL FORMULATION AND DISCUSSION

INDICES:

i = candidate station
 z = zip code under consideration

DATA:

T_{zi} = distance from zip code z to station i
 P_z = population of 17-21 year old males in zip code z
 D_z = population density of zip code z
 NR = number of recruiters in the district
 NS = number of stations to remain open in the district

VARIABLES:

X_i = binary variable (=1 if station i is open and 0 otherwise)
 Y_{zi} = binary variable (=1 if zip code z is assigned to station i and 0 otherwise)
 R_i = number of recruiters to be assigned to station i
 SH_z = proportion of recruiters (or recruiter share) assigned to zip code z

THE LOCATION-ALLOCATION (LOCAL) PROBLEM

OBJECTIVE:

$$\begin{array}{l} \text{MAXIMIZE} \\ SH, R, X, Y, \end{array} \quad \sum_z a * P_z^\alpha * SH_z^\beta * \left(\sum_i \frac{Y_{zi}}{T_{zi}} \right)^\lambda * D_z^\delta$$

CONSTRAINTS:

$$\sum_i X_i = NS \quad (3.1)$$

$$Y_{zi} \leq X_i, \quad \forall z, i \quad (3.2)$$

$$\sum_i Y_{zi} \leq 1, \quad \forall i \quad (3.3)$$

$$\sum_z (Y_{zi} * SH_z) \leq R_i, \quad \forall i \quad (3.4)$$

$$\sum_i R_i \leq NR \quad (3.5)$$

In the above formulation, parameters a , α , β , δ and λ are as defined in Chapter II. The objective is to maximize the total A-Cell contract production. Constraint (3.1) insures that the number of open stations equals the desired number, represented by NS . Constraint (3.2) requires zip codes to be assigned to open stations. Constraint (3.3) allows a zip code to belong to at most one station. However, the inequality allows unproductive zip codes to be left unassigned. Constraint (3.4) requires the recruiter shares distributed among the zip codes in the territory of station i to not exceed the number of recruiters allocated to that station. Finally, constraint (3.5) guarantees that the number of

recruiters assigned to open stations does not surpass the number of recruiters, NR , available for the district.

Since the objective function and constraint (3.4) are nonlinear, LOCAL is a nonlinear mixed integer programming problem which is not a well-solved problem in operations research [Ref. 11]. To solve LOCAL optimally would require a special algorithm to handle both the conditions of nonlinearity and integrality. However, the next chapter describes an alternative solution procedure which produces near optimal solutions efficiently and utilizes currently available software.

IV. A DECOMPOSITION APPROACH FOR THE LOCATION-ALLOCATION PROBLEM

To avoid addressing the integrality and nonlinearity simultaneously in LOCAL, the approach taken in this chapter decomposes the problem into four subproblems, which are solved in sequence. The solution to one subproblem is used as input to the subsequent subproblem. The following are brief descriptions of the subproblems. The sections below describes each of them in detail.

1. The station location subproblem: This subproblem determines the stations to remain open.
2. The recruiter share allocation subproblem: This subproblem assigns (possibly non-integer) numbers of recruiters to the open stations as determined by the first subproblem.
3. The integerization subproblem: When the second subproblem produces non-integer allocations of recruiters to stations, this subproblem optimally rounds those allocations to integer values.
4. The recruiter share reallocation subproblem: Using the allocated recruiters provided by the third subproblem, this subproblem optimally proportions recruiter effort (or recruiter share) to zip codes in the territory of each open station.

A. THE STATION-LOCATION SUBPROBLEM

This subproblem assumes that recruiter shares have already been allocated to each zip code, i.e., the variable SH_i in

LOCAL is fixed to some value, say \bar{SH}_z . Thus, the remaining decision variables are X_i and Y_{zi} which represent the decision to open or close station i and the assignment of zip code z to station i , respectively. This reduces LOCAL to the following station-location subproblem.

STATION-LOCATION SUBPROBLEM

OBJECTIVE:

$$\begin{array}{c} \text{MAXIMIZE} \\ X, Y \end{array} \quad \sum_z a * P_z^a * \bar{SH}_z^\beta * \left(\sum_i \frac{Y_{zi}}{T_{zi}} \right)^\lambda * D_z^\delta$$

CONSTRAINTS:

$$\sum_i X_i = NS \quad (4.1)$$

$$Y_{zi} \leq X_i, \quad \forall z, i \quad (4.2)$$

$$\sum_i Y_{zi} \leq 1, \quad \forall z \quad (4.3)$$

As before, constraint (4.1) insures that the correct number of stations are opened, constraint (4.2) permits a zip code to be assigned only to open stations, and constraint (4.3) guarantees that a zip code is assigned to at most one station.

B. THE RECRUITER SHARE ALLOCATION SUBPROBLEM

An optimal solution to the station location subproblem, (X^*, Y^*) , specifies the stations to remain open and the assignment of zip codes to stations. This provides a new alignment of recruiting stations within the district based

upon previously fixed levels of recruiter shares, \bar{SH}_z .

However, this allocation of recruiter share may not be optimal or feasible under the new alignment. For example, it would be infeasible for a zip code that was not assigned to any station to be allocated a recruiter share. The following nonlinear program optimally allocates recruiter shares to zip codes in the territory of an open station using the station alignment given by X' and Y' .

RECRUITER SHARE ALLOCATION SUBPROBLEM

OBJECTIVE:

$$\text{MAXIMIZE}_{SH} \sum_z a * P_z^\alpha * SH_z^\beta * \omega_z^\lambda * D_z^\delta$$

CONSTRAINTS:

$$\sum_i \sum_z Y_{zi}' * SH_z \leq NR \quad (4.4)$$

In the objective function, $\omega_z = Y_{iz}' / T_{iz}$ where T_{iz} is defined to be distance from zip code z to station i . Constraint (4.4) prevents the recruiter shares distributed among zip codes from exceeding the available recruiters. The subproblem is an easy nonlinear programming problem since the recruiter share variable, SH_z , is allowed to vary continuously for all z and there is only one constraint. The objective function is strictly concave because $\beta < 1$. This guarantees solutions to the subproblem to be globally optimal.

C. THE INTEGERIZATION SUBPROBLEM

Let SH_z^* denote an optimal solution to the recruiter share allocation subproblem in the previous section. Then, the number of recruiters to be assigned to station i is:

$$R_i = \sum_z Y_{zi}^* SH_z^* \quad \forall i. \quad (4.5)$$

These R_i values may not be integer. Simple rules for rounding are insufficient for they may not be feasible. One feasible method of rounding R_i is due to Rosenthal [Ref. 12].

He first assigns \bar{R}_i recruiters to station i , where

$$\bar{R}_i = \text{FLOOR}(R_i). \quad (4.6)$$

This means that R_{res} , defined below, is the number

$$R_{res} = NR - \sum_i \bar{R}_i \quad (4.7)$$

of recruiters left unassigned. Then, Rosenthal's method allocates these residual recruiters to stations so as to maximize their marginal contract production. Note that R_{res} is integer since both NR and \bar{R}_i are integer. Since, R_{res} is

expected to be much smaller than NR for practical problems, the degradation of the objective function from the continuous solution is very small.

The marginal productivity from adding u_i recruiters to station i is the difference between the expected A-Cell contract production with $(\bar{R}_i + u_i)$ recruiters and the production with only \bar{R}_i recruiters. The form of the NRC regression function (see Model 1 on page 8) does not permit a direct calculation of the marginal productivity, so the following approximation is used instead.

Define Ψ_i to be the set of all zip codes assigned to station i . Then, using the same notation as before, the productivity of station i is:

$$Q_i = \sum_{z \in \Psi_i} c_z SH_z^\beta \quad (4.8)$$

$$\text{where } c_z = a P_z^\alpha \omega_z^\lambda D_z^\delta.$$

Now suppose the recruiter share, SH_z^* , allocated to zip code z is proportional to the ratio of the target population in each zip code over the entire population in its affiliated station's territory. Then, the recruiter share for zip code z is defined to be:

$$SH_z = R_i \frac{P_z}{\sum_{z' \in \psi_i} P_{z'}} \quad \text{for all } z \in \psi_i. \quad (4.9)$$

Using equation (4.9), the productivity for station i can be rewritten as

$$Q_i = \sum_{z \in \psi_i} c_z \left(R_i \frac{P_z}{\sum_{z' \in \psi_i} P_{z'}} \right)^\beta \quad (4.10)$$

or equivalently

$$Q_i = R_i^\beta \sum_{z \in \psi_i} K_z \quad \text{where } K_z = c_z \left(\frac{P_z}{\sum_{z' \in \psi_i} P_{z'}} \right)^\beta. \quad (4.11)$$

As defined, K_z is constant for each zip code z . Thus, the marginal productivity for u_i additional recruiters assigned to station i is:

$$MQ_i = \{(R_i + u_i)^\beta - R_i^\beta\} \sum_{z \in \psi_i} K_z. \quad (4.12)$$

Using this marginal productivity, the integerization subproblem can be formulated as follows:

ADDITIONAL INDICES:

j = possible number of residual recruiters assigned to a station

ADDITIONAL VARIABLES:

b_{ij} = binary variable (=1 if j residual recruiters are assigned to station i and 0 otherwise)

INTEGERIZATION SUBPROBLEM

OBJECTIVE:

$$\text{MAXIMIZE}_{\mathbf{b}} \quad \sum_i \sum_j \left[b_{ij} * \sum_{z \in \Psi_i} K_z * \{ (R_i + j)^\beta - R_i^\beta \} \right]$$

CONSTRAINTS:

$$\sum_i \sum_j (j * b_{ij}) \leq R_{res} \quad (4.13)$$

$$\sum_j b_{ij} \leq 1, \quad \forall i \quad (4.14)$$

In constraint (4.13), the inner summation represents the number of additional recruiters to be assigned to station i and the constraint (4.13) itself insures that the additional recruiters assigned to all stations does not exceed the number of residual recruiters, R_{res} . Constraint, (4.14), guarantees that each station receives a unique number of additional recruiters, which can possibly be zero. Given an optimal solution, b_{ij}^* ,

$$u_i^* = \sum_j j * b_{ij}^* \quad (4.15)$$

is the additional recruiters for station i ,

i.e., $R_i^* = \bar{R}_i + u_i^*$.

D. THE RECRUITER SHARE REALLOCATION SUBPROBLEM

Solution to the integerization problem, R_i^* , results in new recruiter allocations to each station i when the previous allocations were noninteger. Thus, the distribution of recruiter shares from the recruiter share allocation subproblem may not match the number of recruiters at each station after the integerization. So, the *recruiter share reallocation* subproblem is to optimally reallocate recruiter shares to each covered zip code, given an integer assignment of recruiters to stations. Mathematically, this problem is formulated with the same variables and objective function as the recruiter share allocation subproblem, but constraint (4.4) is replaced by the following:

$$\sum_z (Y_{zi}^* * SH_z) \leq R_i^* \quad \forall i. \quad (4.16)$$

This prevents the recruiter shares distributed to zip codes in the territory of station i from exceeding the number of recruiters allocated to that station as specified by the integerization subproblem.

V. RESULTS FROM THE DECOMPOSITION APPROACH

The principal focus of this chapter is to present the results for the Boston Recruiting District using the decomposition approach. However, first it is necessary to develop a bounding technique which can verify that the approach produces solutions with acceptable quality.

A. BOUNDING THE DECOMPOSITION APPROACH

Solutions obtained from the decomposition process are guaranteed to be both integer and feasible. To demonstrate the quality of the decomposition approach, its solutions are compared against a known upper bound to the LOCAL problem. Such a bound is determined by optimally solving a nonlinear programming relaxation (NLPR) of the LOCAL problem.

The NLPR is obtained by omitting all binary requirements, i.e., the binary variables are allowed to range continuously in the interval $[0,1]$. Mathematically, the problem is formulated in the same manner as the LOCAL problem with the exceptions that constraints (3.4) and (3.5) are replaced by the following linear constraints:

$$\sum_z SH_z \leq NR \quad (5.1)$$

$$SH_z \leq \pi * \sum_i Y_{zi}, \quad \forall z. \quad (5.2)$$

Constraint (5.1) insures the recruiter shares distributed to zip codes do not exceed the available number of recruiters. Constraint (5.2) guarantees recruiter shares are distributed only to those zip codes assigned to stations; the parameter π is chosen to be large enough so that sufficient recruiter share can be assigned to a given zip code. With no binary restrictions, all of the decision variables are continuous and, as shown in Chapter II, the objective function is strictly concave. Consequently, the solution to the NLPR is unique and globally optimal [Ref. 13].

Table 1, on the following page, compares the results from the decomposition process against the corresponding upper bound for twenty-four problems using the 1991 recruiting data. Each row of the table specifies the district, number of stations and recruiters for the problem. Additionally, the values of recruiters and stations as a percentage of the 1991 alignment is given within the parentheses. The percentage by which solutions from the decomposition approach differ from the upper bound is displayed in the last column.

Table 1 shows that the decomposition process underestimates the true optimal solutions by at most 5.19 percent and on average 2.5 percent. This difference is acceptable since the A-Cell production is predicted by a regression model with $R^2 = 0.5303$. Using the Amdahl 5990-500 Computer at Naval Postgraduate School, the average solution

TABLE 1: QUALITY OF SOLUTIONS FROM THE DECOMPOSITION APPROACH

District	Stations	Recruiters	Total A-Cells		% Difference
			NLPR	Decomp.	
161	6 (23.1%)	13 (24.1%)	253.61	242.85	4.24
161	13 (50.0%)	27 (50.0%)	355.77	348.85	1.95
161	19 (73.1%)	40 (74.1%)	422.22	418.70	0.84
161	26 (100%)	54 (100%)	478.75	477.60	0.24
101	6 (22.2%)	18 (24.7%)	275.51	261.55	5.07
101	13 (48.1%)	36 (49.3%)	388.65	378.10	2.71
101	20 (74.1%)	54 (74.0%)	470.34	464.59	1.22
101	27 (100%)	73 (100%)	535.54	534.76	0.15
104	9 (23.7%)	29 (25.0%)	420.60	412.17	2.00
104	19 (50.0%)	58 (50.0%)	570.78	564.04	1.18
104	28 (73.7%)	87 (75.0%)	674.94	672.09	0.42
104	38 (100%)	116 (100%)	758.53	757.64	0.12
103	7 (25.0%)	26 (25.0%)	297.52	282.09	5.19
103	14 (50.0%)	52 (50.0%)	414.23	403.67	2.55
103	21 (75.0%)	78 (75.0%)	498.84	495.57	0.65
103	28 (100%)	104 (100%)	564.62	564.34	0.05
106	6 (23.1%)	20 (25.0%)	248.26	233.85	5.81
106	13 (50.0%)	40 (50.0%)	355.33	344.32	3.10
106	19 (73.1%)	60 (75.0%)	427.43	421.55	1.38
106	26 (100%)	80 (100%)	486.65	486.63	0.00
119	6 (22.2%)	19 (24.1%)	280.86	273.31	2.69
119	13 (48.1%)	39 (49.4%)	389.89	382.42	1.92
119	20 (74.1%)	59 (74.7%)	467.60	464.71	0.62
119	27 (100%)	79 (100%)	527.78	527.07	0.13

time for the twenty-four problems using the decomposition process is approximately 286 CPU seconds, while for the NLPR the time is approximately 19,275 CPU seconds. Thus, not only does the decomposition process produce near optimal integer solutions, but the solutions are obtained reasonably fast.

Further analysis of Table 1 reveals that the numbers of recruiters and stations for which the decomposition process produces acceptable solutions ranges from approximately 25 to 100 percent of the 1991 alignment. Moreover, the solution quality of the decomposition approach improves as the numbers of recruiters and stations are near 100 percent of the 1991 alignment. To demonstrate, when the number of recruiters and stations are approximately 25 percent of the 1991 level, the average difference is 3.45 percent. When they are approximately 50, 75 and 100 percent of the 1991 level, the average differences are 1.92 percent, 0.73 percent and 0.12 percent, respectively. This is due in part to the fact that the relaxed model, NLPR, yields more variables with integer values for problems with larger numbers of recruiters and stations.

B. RESULTS AND ANALYSIS

The four optimization problems were implemented in the General Algebraic Modeling System [Ref. 14] (GAMS) and the resulting integer and nonlinear programming problems were solved by the XS [Ref. 15] and MINOS

[Ref. 16] solvers, respectively. The 1991 recruiting data for the Boston district is used to demonstrate applications of the model. For this example, the list of candidate stations consist of only those that were operational in 1991.

In 1991, the Boston district had 42 recruiting stations and 111 recruiters. Figure 2 depicts results from the decomposition process with the number of stations varied from 10 to 42 and the number of recruiters from 27 to 111. The graphs in this figure show how the number of A-Cells increase as a function of the number of stations and recruiters.

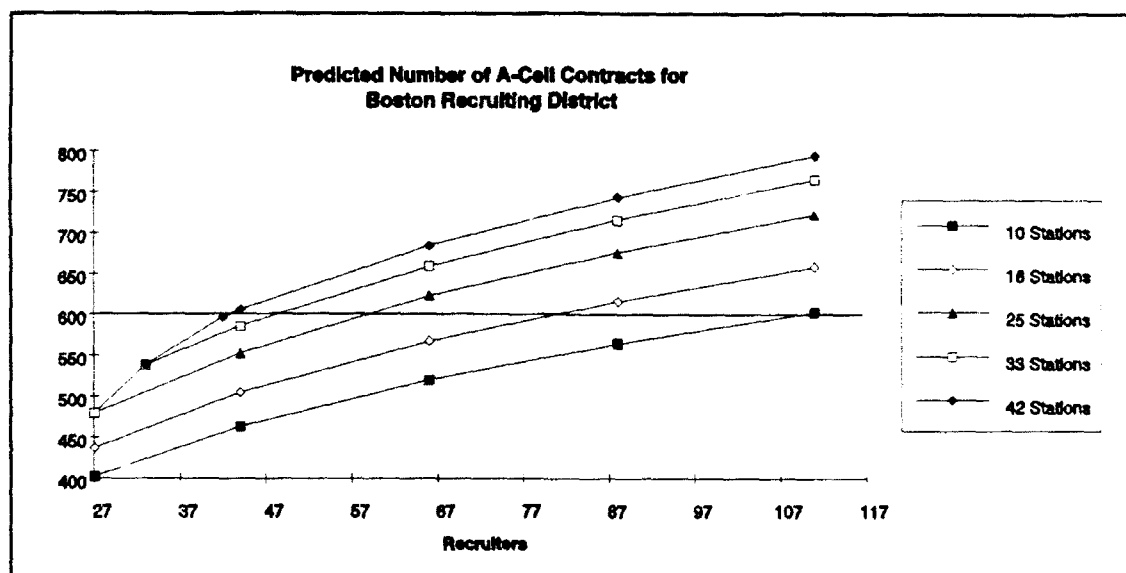


Figure 2: Predicted Number of A-Cell Contracts for Various Numbers of Recruiters and Stations for Boston District

However, the expected gain due to the increase in the number of stations is decreasing. For example, the gain in the number of A-Cells by increasing the number of stations from 16 to 25 is greater than that from either 25 to 33 or 33 to 42.

Information in Figure 2 can be used to construct isoquant curves representing combinations of stations and recruiters that yield the same number of A-Cell contracts. For example, the points where the horizontal line in Figure 2 intersects the five graphs represent combinations of recruiters and stations that produce an expected 600 A-Cell contracts for the Boston district. When several isoquants are constructed, they can be graphically displayed as in Figure 3.

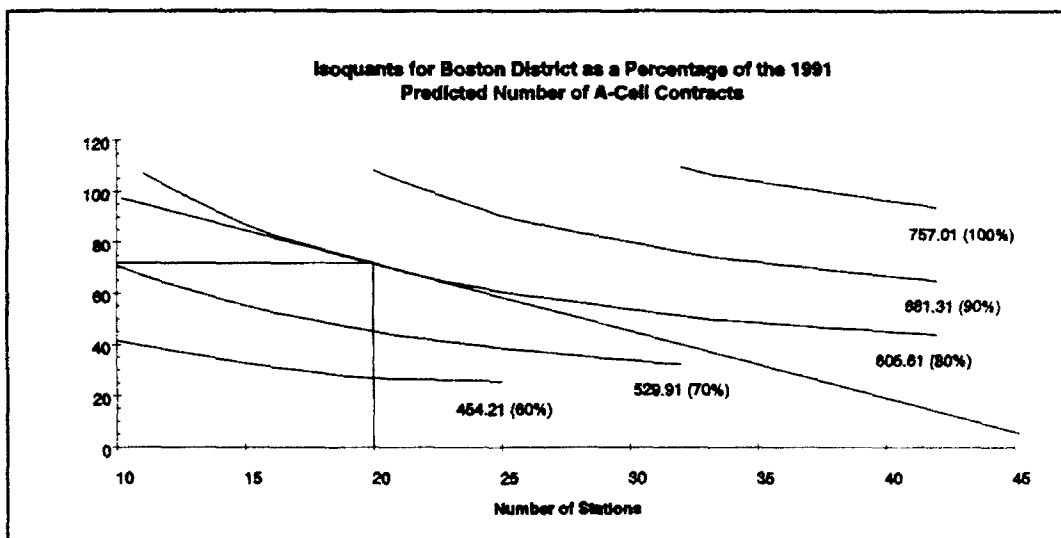


Figure 3: Isoquants of Predicted Number of A-Cell Contracts for Boston District and Isocost Line

To illustrate the use of the isoquants, suppose that the annual operating budget for Boston district, the average cost per recruiter and the average cost per station are \$1,000K, \$7.93K and 21.47K, respectively. Then the isocost line specified by the equation,

$7.93 \times (\text{no. of recruiters}) + 21.47 \times (\text{no. of stations}) = 1000$,
represents all possible combinations of stations and

recruiters that satisfy the \$1,000K budget exactly. This isocost line corresponds to the straight line shown in Figure 3. Since this line is tangential to the isoquant for approximately 605.61 A-Cells, this is the maximum expected number of A-Cells obtainable with \$1,000K. To obtain this number of A-Cells, Figure 3 shows that 20 stations and 72 recruiters are necessary. On the following page, Table 2 displays the stations that should remain open with 72 recruiters. Each row contains the station identification number and the number of recruiters that should be assigned to that station. Note the total A-Cells in Table 2 is 609.03 instead of 605.61. This difference is due to the interpolation used in constructing the isoquants in Figure 3.

By varying the annual operating budget, different isocost lines can be drawn. Each would be tangent to different isoquants thereby yielding different combinations of recruiters and stations for the different budget limits. In this manner, one can easily obtain the optimal numbers of recruiters and stations for any operating budget.

TABLE 2: RECOMMENDED ALLOCATION OF 72 RECRUITERS IN BOSTON DISTRICT WITH 20 STATIONS REMAINING OPEN

Station I.D.	No. Recruiters	Expected A-Cells
10	4	33.35
30	4	31.00
50	5	41.71
60	4	34.11
65	3	24.22
90	3	24.17
100	3	25.63
130	4	36.05
145	5	41.95
160	4	33.39
180	4	34.64
220	3	22.67
230	3	26.20
240	1	9.12
250	6	50.90
270	3	28.40
310	3	27.55
340	3	23.65
344	3	28.02
346	4	32.30
Totals	72	609.03

VI. ALLOCATING RECRUITERS AND STATIONS AMONG DISTRICTS

The problem addressed in this chapter assumes that the current operating budget can support given numbers of stations and recruiters. These stations and recruiters are to be distributed among a collection of districts, such as a recruiting area. Intuitively, districts that produce more A-Cell contracts should receive more stations and recruiters. The overall objective is to maximize the total A-Cell contract production for the collection of districts. This problem is referred to as the multidistrict allocation (MDA) problem and can be approached in two ways. One possibility is to consider the collection of two or more districts as one 'super' district and use the method described in previous chapters to solve it. The other is to formulate the problem as a nonlinear resource allocation problem.

The first approach has the disadvantage that the number of variables would be too large for solution in a reasonable time. For example, Recruiting Area 1, consisting of the following districts: Albany, Boston, Buffalo, New York, Harrisburg, Philadelphia, Pittsburgh and New Jersey, contained 248 stations and 5,458 zip codes in 1991. Counting the binary decision variables alone, the problem of allocating recruiters and stations to districts in Area 1 can contain up to $248 + (248 \times 5,458)$ or 1,353,832 variables. Thus, the resource

allocation formulation, which has a much smaller number of variables, is chosen instead. The difficulty with this formulation is that the objective function is not expressible in closed form. In one of the sections below, the objective function is approximated using piecewise linear functions. Other sections formally state the formulation, describe an implementation and present an example.

A. THE MULTIDISTRICT RESOURCE ALLOCATION FORMULATION

In this allocation problem, a number of recruiters and stations are to be distributed to two or more districts so as to maximize the total A-Cell production. Given that there are costs associated with employing a recruiter and maintaining a station, there is also a constraint to limit the cost not to exceed a certain budget. Below is a formulation of this problem.

MULTIDISTRICT ALLOCATION (MDA) PROBLEM

INDICES:

d = district

DATA:

B = total annual budget for the districts considered

NR = number of recruiters to be allocated

NS = number of stations to be allocated

C_d^R = average annual cost for a recruiter in district d

$C_d^S =$ average annual cost of a station in district d

DECISION VARIABLES:

$R_d =$ number of recruiters assigned to district d

$S_d =$ number of stations assigned to district d

OBJECTIVE:

$$\text{MAXIMIZE}_{R, S} \quad \sum_d F_d(R_d, S_d)$$

CONSTRAINTS:

$$\sum_d R_d \leq NR \quad (6.1)$$

$$\sum_d S_d \leq NS \quad (6.2)$$

$$\sum_d (C_d^R R_d + C_d^S S_d) \leq B \quad (6.3)$$

In the objective, $F_d(r, s)$ represents the maximum A-Cell production using r recruiters and s stations in district d . Since $F_d(r, s)$ is generally nonlinear, the above problem is a nonlinear integer programming problem. Its first two constraints, (6.1) and (6.2), insure that the number of assigned recruiters and stations do not exceed the available number and the last, constraint (6.3), ensures that the cost does not exceed the budget.

The function $F_d(r,s)$ is not expressible in closed form. To evaluate $F_d(r,s)$ at a particular value of r and s , the LOCAL problem corresponding to district d must be solved. As seen in Chapter V, when all the variables in LOCAL are allowed to vary continuously, the resulting problem is a strictly concave programming problem. The function $F_d(r,s)$ is known as an optimal value function and can be shown to be concave [Ref. 13]. When some of the variables in LOCAL are restricted to integers, the concavity for $F_d(r,s)$ is not guaranteed. However, our experiments empirically support the assumption that $F_d(r,s)$ is concave when s is integer and r is allowed to range continuously.

Based on the magnitudes of R_d and S_d , the integer restriction of R_d may be relaxed, but not S_d . Using this form of relaxation, $F_d(r,s)$ can be approximated with piecewise linear functions and the MDA problem can be reformulated as a linear integer programming problem. The next section describes the approximation scheme and the linear formulation.

B. APPROXIMATING THE MULTIDISTRICT ALLOCATION PROBLEM

Let $f_d(r)$ denote the value of $F_d(r,s)$ at a fixed value of s . Since it is assumed that noninteger recruiters are allowed, $f_d(r)$ is a continuous function of r and can be approximated with a piecewise linear function $g_d(r)$.

By virtue of the assumption that $\tau_d(r)$ is piecewise linear and concave, its value can be maximized by linear programming

[Ref. 17]. As an illustration, suppose $g_d(r)$ is specified by the points $\{u_i, g_d(u_i)\}$ for $i = 1, \dots, 4$ (see Figure 4) where $g_d(u_i)$ represents an actual solution to the LOCAL problem with u_i recruiters allocated to s stations in district d .

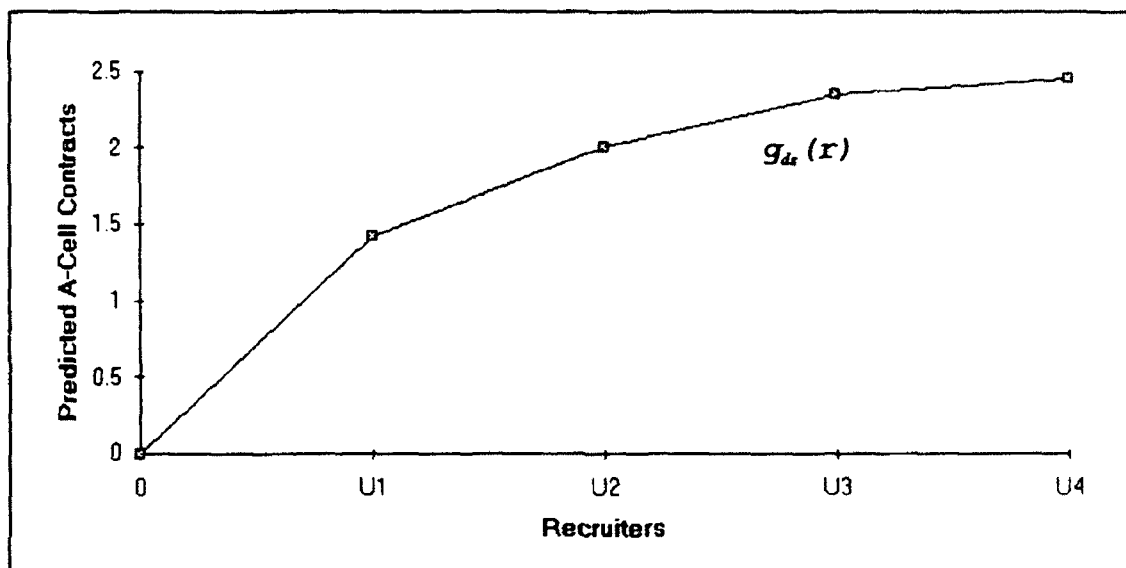


Figure 4: A Piecewise Linear Concave Function

Let p_i be the proportion of the recruiters r represented in the interval between u_{i-1} and u_i where $u_i > u_{i-1}$. Then, any $0 \leq r \leq u_4$ can be written as

$$r = \sum_{i=1}^4 p_i, \quad \forall p_i \leq u_i - u_{i-1} \quad (6.4)$$

where p_i is nonnegative and $p_{i+1} > 0$ only when $p_i = u_i$. For example, let $\{u_1, u_2, u_3, u_4\} = \{10, 20, 30, 40\}$, then for $r = 25$, $\{p_1, p_2, p_3, p_4\} = \{10, 10, 5, 0\}$. Thus, by defining r in this manner, $g_{d,s}(r)$ can be written as

$$g_{d,s}(r) = \sum_{i=1}^4 m_i p_i \quad (6.5)$$

where m_i represents the slope of the piecewise linear segment between u_{i-1} and u_i .

In the formulation below, the approximating function $g_d(r)$ is not computed for every possible value of s . Instead, $g_d(r)$ is computed for only a small collection of s values. The remaining $g_d(r)$ are obtained by linear interpolation. For example, assume that $g_{d,10}(r)$ and $g_{d,15}(r)$ have been computed, then $g_{d,12}(r)$ is approximated as follows:

$$g_{d,12}(r) = \frac{3}{5}g_{d,10}(r) + \frac{2}{5}g_{d,15}(r). \quad (6.6)$$

Mathematically, the approximation to the MDA problem is formulated below.

APPROXIMATE MULTIDISTRICT ALLOCATION PROBLEM

INDICES:

- d = district
- s = number of stations to remain open
- k = linear segment for the piecewise linear functions

GIVEN AND DERIVED DATA:

- C_d^S = average annual cost of a station in district d
- C_d^R = average annual cost for a recruiter in district d
- B = total annual budget
- M_{dk} = the slope of segment k for the piecewise linear function approximating $f_d(r)$
- U_{dk} = upper bound for the interval of segment k for district d
- SU_d = upper limit on the number of stations which may remain open within district d
- SL_d = lower limit on the number of stations which may remain open within district d
- NS = number of stations available
- NR = number of recruiters available

VARIABLES:

- p_{dsk} = number of recruiters in the interval for segment k with s stations open in district d
- q_{ds} = binary variable(=1 if s stations are to remain open in district d and 0 otherwise)
- v_d = slack variable

OBJECTIVE:

$$\text{MAXIMIZE}_{p, q} \sum_d \sum_s \sum_k M_{dsk} * p_{dsk}$$

CONSTRAINTS:

$$p_{dsk} \leq U_{dk} * q_{ds}, \quad \forall d, s, k \quad (6.6)$$

$$\sum_s q_{ds} = 1, \quad \forall d \quad (6.7)$$

$$\sum_d \sum_s (s * q_{ds}) \leq NS \quad (6.8)$$

$$\sum_d \sum_s \sum_k p_{dsk} \leq NR \quad (6.9)$$

$$v_d + \sum_s (s * q_{ds}) = SU_d, \quad \forall d \quad (6.10)$$

$$s * q_{ds} \leq \sum_k p_{dsk}, \quad \forall d, s \quad (6.11)$$

$$\sum_d \sum_s \sum_k (p_{dsk} * C_d^R) + \sum_d \sum_s (q_{ds} * C_d^S) \leq B \quad (6.12)$$

In the above formulation, constraint (6.6) ensures that the correct segments of the piecewise linear functions are used for each district. Constraint (6.7) ensures a unique number of stations remain open within each district. Constraint (6.8) allows at most NS stations to remain open. Constraint (6.9) insures the number of recruiters to be allocated, NR , do not exceed the number available.

Constraint (6.10) is a two-sided constraint represented by a single constraint [Ref. 18]. It guarantees that the number of stations open in each district is between a

lower and an upper bound. A lower bound is established to avoid potential political problems by not recruiting within specific districts or conversely recruiting exclusively in others. The upper bound prevents the number of stations within any district from exceeding the 1991 level, since NRC is downsizing.

Constraint (6.11) guarantees that the number of recruiters assigned to any district is at least as large as the number of stations. Finally, constraint (6.12) insures the costs associated with the number of recruiters and stations allocated do not exceed the available annual budget.

C. EXAMPLE

To illustrate, the above formulation was implemented in GAMS and solved by OSL [Ref. 19]. The data inputs are from Recruiting Area 1 which, as mentioned previously, consists of eight districts. In 1991, Area 1 had 248 station to which 702 recruiters were assigned. NRC estimated that the average annual costs for recruiters and stations for all districts in Area 1 are approximately \$50K and \$9.472K, respectively. Using these estimates, the 1991 operating budget for Area 1 is calculated as

$$(248 \times \$9.472K) + (702 \times \$50K) = \$37,449K.$$

On the following page, Figure 5 depicts solutions to the IA problem for budget limits ranging from 50-90 percent of the 1991 budget. To obtain the graph in this figure, a total of

five multidistrict problems must be solved which takes on the average 3.655 CPU seconds using the Amdahl Computer. The graph in this figure shows how the number of A-Cell contracts increase as a function of the operating budget.

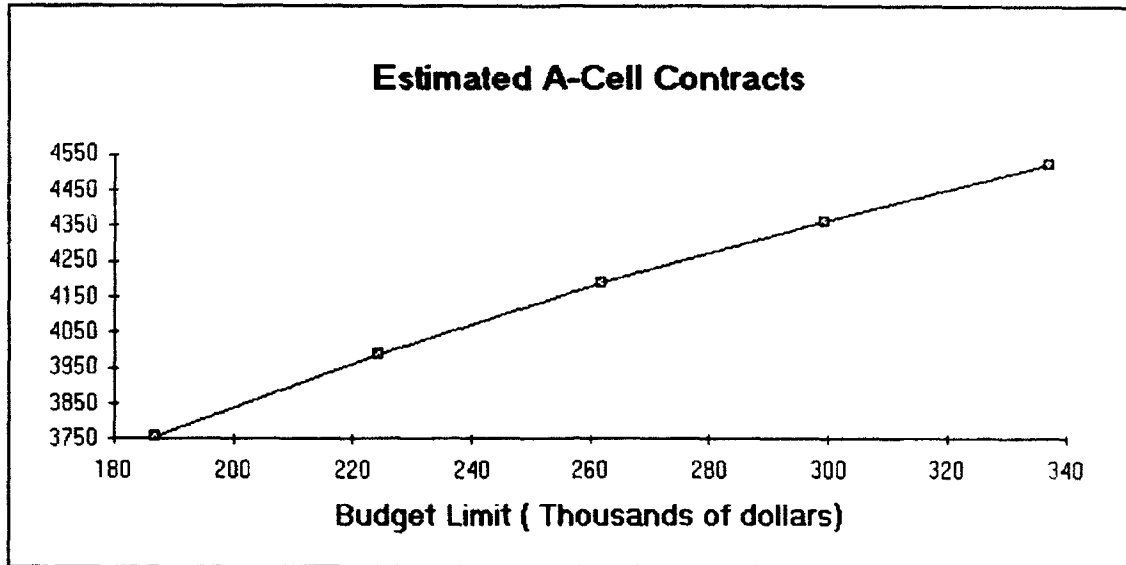


Figure 5: Solutions to the MDA problem for Area 1

Based upon the graph in Figure 5, the increase in A-Cells appears to be fairly constant with respect to budget increases. This is because it is more cost effective to increase the number of recruiters instead of opening stations as the budget increases.

In particular, consider a ten percent reduction in the budget, i.e., solve the multidistrict allocation problem for Area 1 with \$33,699K instead of \$37,449K. This yields a new alignment displayed in Table 4. When compared to the A-Cell contracts produced under the 1991 alignment (i.e., 4,432

contracts), the new one yields more contracts by approximately two percent.

TABLE 4: A SOLUTION TO THE MDA PROBLEM FOR AREA 1

Districts	Number of Recruiters	Number of Stations	Estimated Cost (in thousands of \$)	Predicted A-Cell Contracts
101	73	27	3905.77	535.54
102	111	42	5947.86	796.10
103	62	28	3365.24	487.60
104	116	38	6159.97	758.53
106	64	26	3446.29	457.49
119	79	27	4205.77	527.78
120	68	34	3722.08	480.41
161	54	26	2946.29	478.74
Totals	627	248	33699.27	4522.19

VII. SUMMARY AND CONCLUSIONS

This thesis addresses two problems in aligning the recruiting structure for Navy Recruiting Command. The first problem involves two decisions affecting recruiting stations within a single recruiting district: which stations should remain open and how many recruiters to assign to each open station? The second problem is to decide how many recruiters and stations each district should have.

The first problem is modeled as a nonlinear mixed integer programming problem. To obtain a solution with readily available software, the problem is decomposed into four subproblems solved sequentially. They are (i) the station location, (ii) the recruiter share allocation, (iii) the integerization and (iv) the recruiter share reallocation problems. This decomposition approach produces good integer solutions relatively fast. These solutions are within 2.5 percent of optimality on the average and within 5.19 percent in the worst case. Results from the decomposition approach were used to construct isoquants for A-Cell production in the Boston district. When superimposed with an isocost line, these isoquants identify the most economical number of recruiters and stations for the Boston district.

To keep the number of variables at a manageable size, the second problem is modeled as a nonlinear resource allocation

problem in which the objective function is not available in closed form. To efficiently solve this problem, the function is approximated in a piecewise linear fashion using the results from the first problem. The approximation yields a linear integer programming problem which can be solved by readily available software. For a given operating budget, this integer programming problem was used to produce optimal numbers of recruiters and stations to be allocated to each district in Recruiting Area 1. Using these results, it is shown that the 1991 A-Cell production level in Area 1 can be maintained using only 90 percent of the 1991 operating budget.

In conclusion, this thesis provides NRC with a tool to aid in their current downsizing efforts. The effectiveness of this tool largely depends on two unique methodologies developed in earlier chapters. One utilizes decomposition to obtain solutions to nonlinear mixed integer programming problems for which no readily available algorithm exists. Unlike other heuristic approaches, the decomposition produces answers that can be considered optimal for all practical purposes. The other methodology uses an alternate formulation and an approximation scheme to solve problems that are otherwise too large for currently available solvers.

A. AREAS FOR FURTHER RESEARCH

The results of this thesis also point out the following areas for further research.

1. Robustness of the Model Against Different Forecasting Models for Contract Production

This thesis uses the log-linear form of the forecasting model as recommended by NRC. However, there are other competing forecasting models which are of different forms and include other demographic and socio-economic factors. If different forecasting models generate drastically different station locations and recruiter allocations, then the choice of forecasting models will play an important role in the realignment decisions. However, if different forecasting models generate similar locations and allocations, then the forecasting model which expedites the solution process should be chosen. In any case, this discussion highlights the need for further investigation to determine model robustness.

2. Realignment System Development

It is inevitable that the need to realign the recruiting organization will re-occur in the future. This may be due to a change in policy or a shift in the location of the target market. To keep recruiting efficient, the recruiting organization should be responsive to these changes. This will require that the optimization models developed in this thesis be integrated in a user-friendly system.

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